

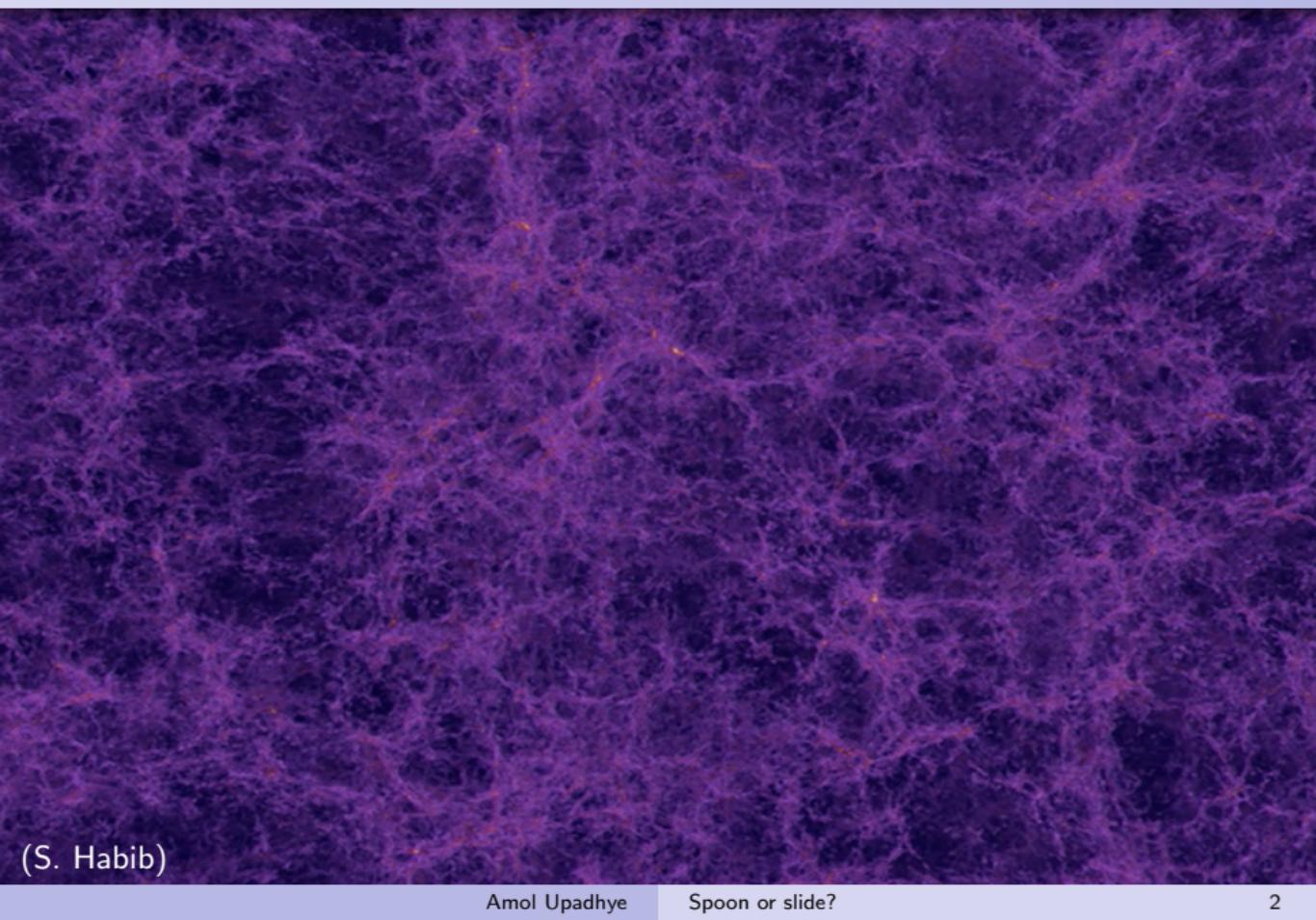
Spoon or slide?

The non-linear matter power spectrum in the presence of massive neutrinos

Hannestad, Upadhye, and Wong,
JCAP 11 (2020) 062, arXiv:2006.04995

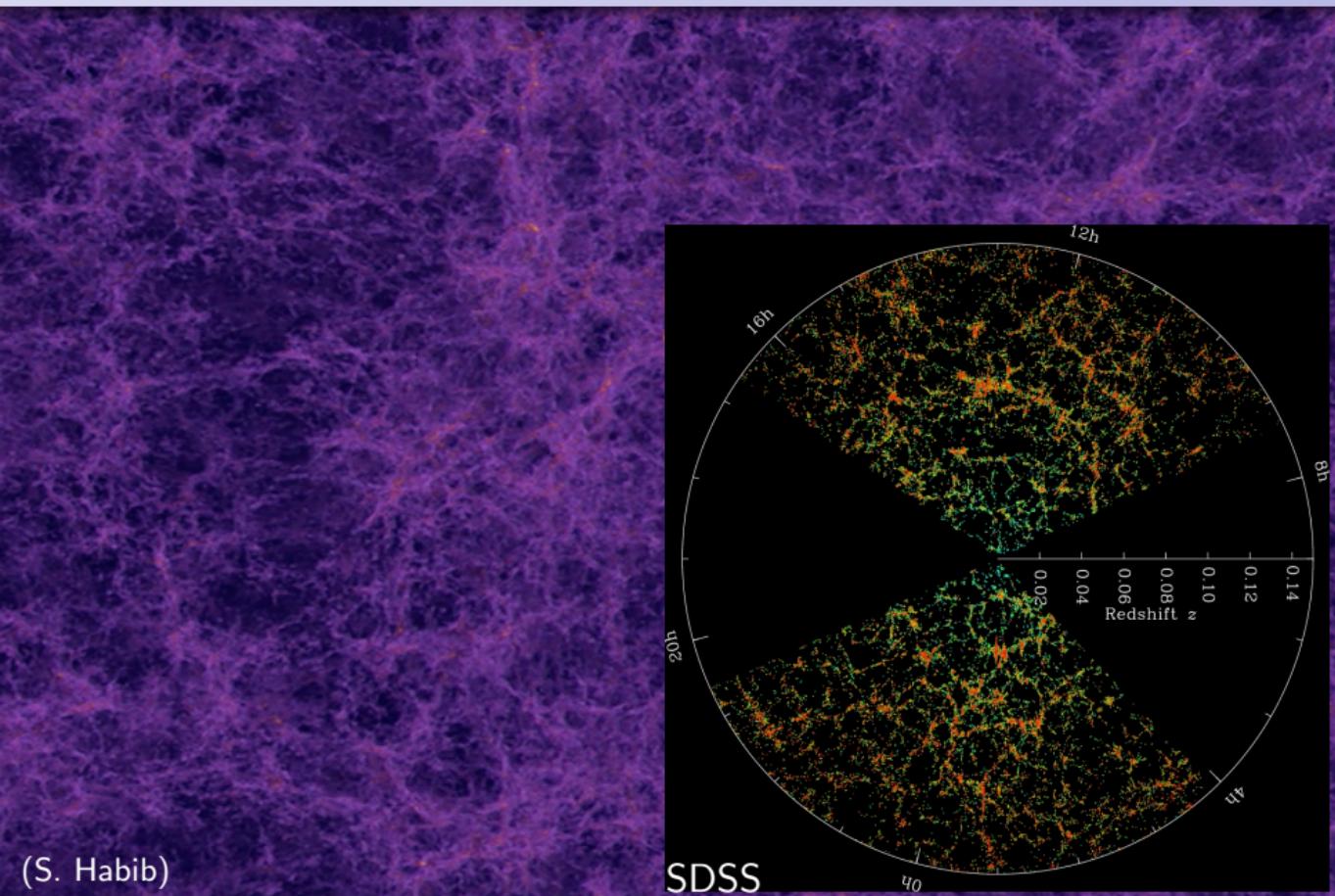
Amol Upadhye
SWIFAR, YNU
Journal Club
November 8, 2023

Large-scale cosmological structure



(S. Habib)

Large-scale cosmological structure



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Amol Upadhye

SDSS

Spoon or slide?

Large-scale cosmological structure

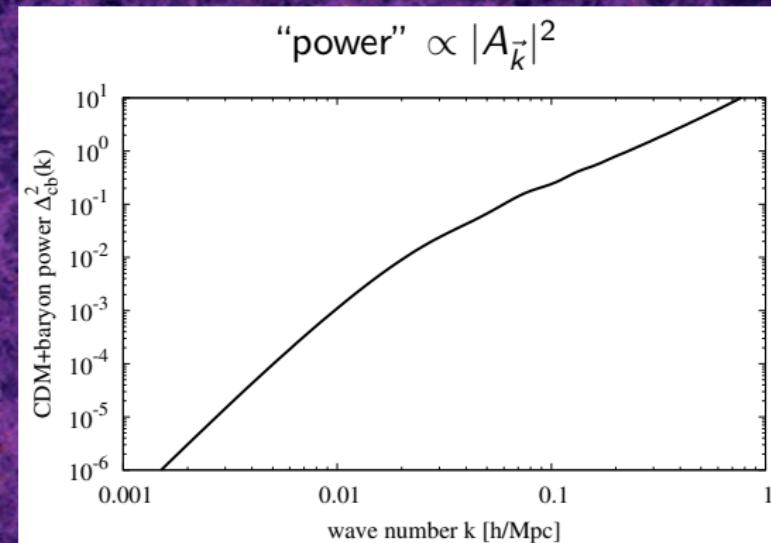


$$\delta \rho / \rho = A_{\vec{k}_2} \exp(\vec{k}_2 \cdot \vec{x})$$

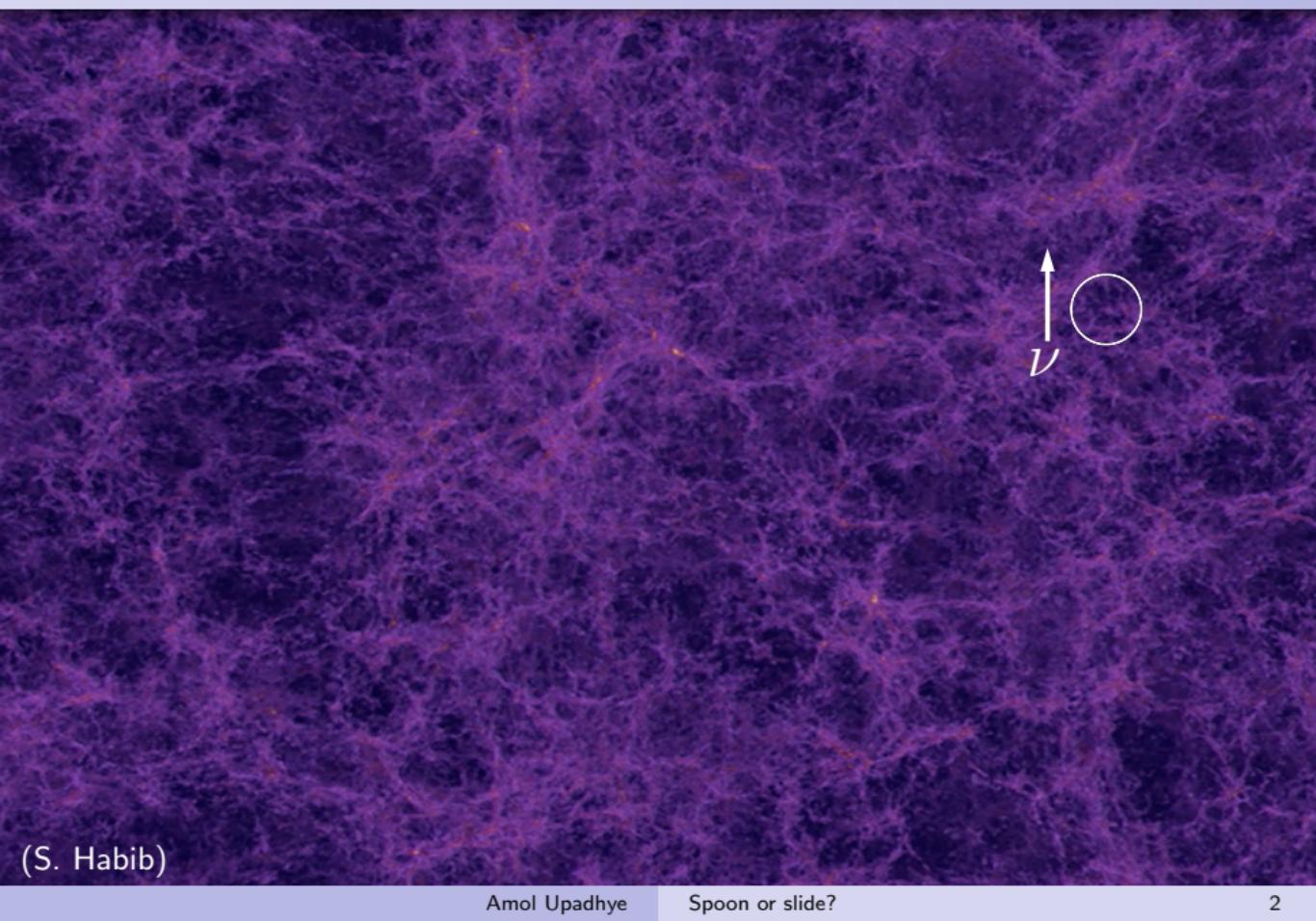
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(S. Habib)

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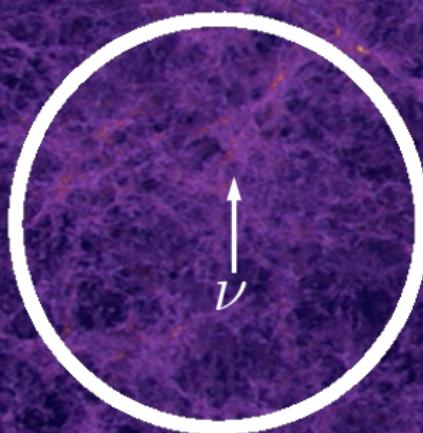


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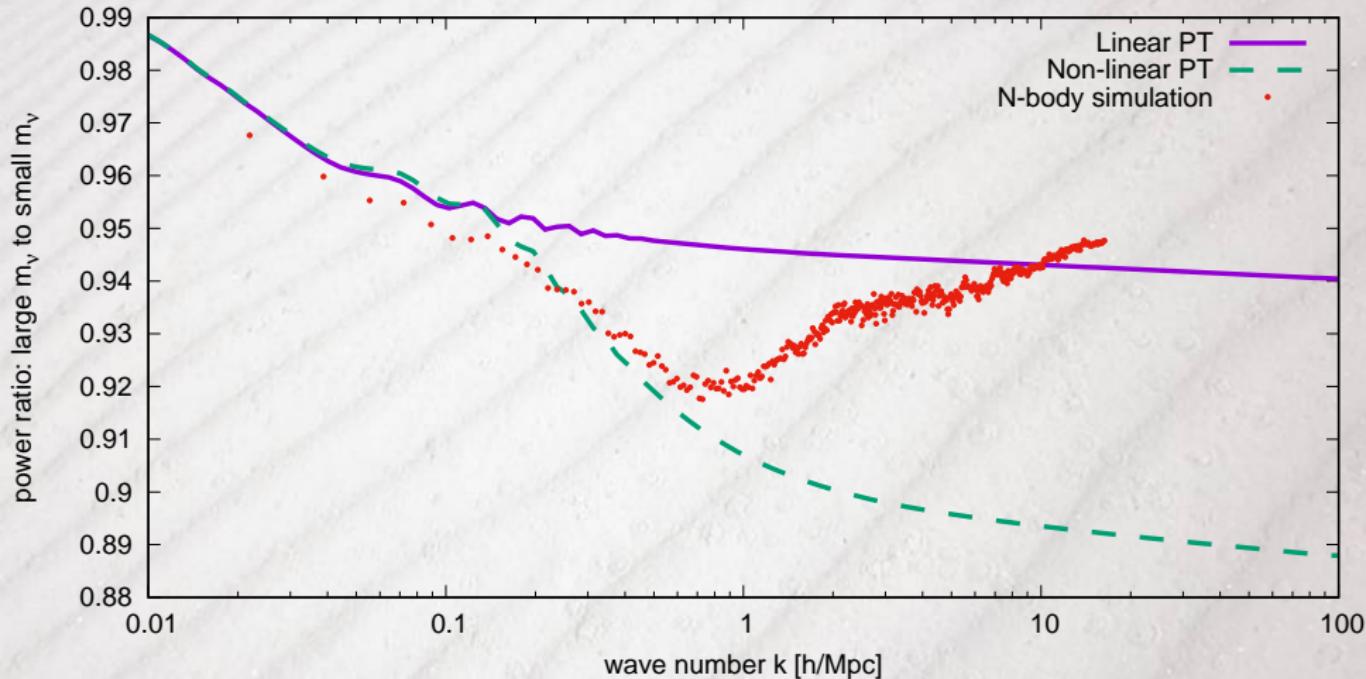


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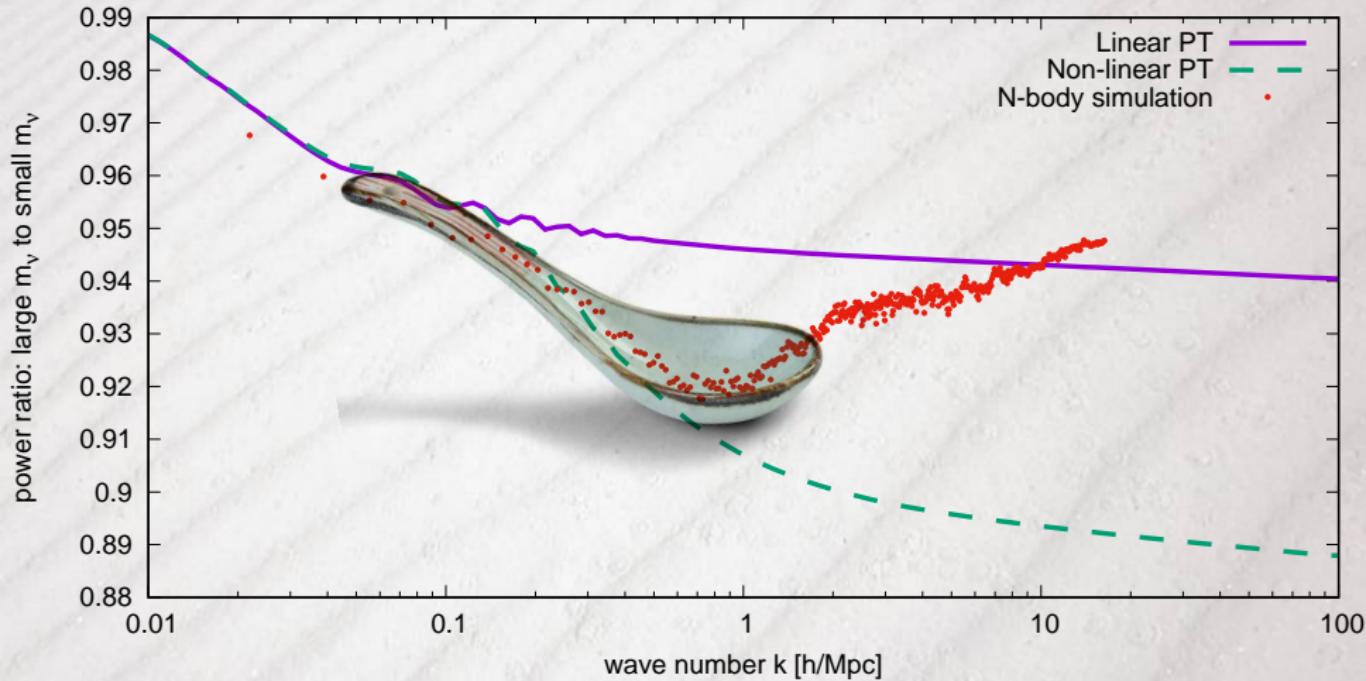
Large-scale cosmological structure



Neutrino suppression: spoon or slide?



Neutrino suppression: spoon or slide?



Halo model: beyond perturbation theory

The halo model of large-scale structure assumes that all Cold Dark Matter (CDM) and baryons is in collapsed objects called halos.

It has three ingredients:

- ➊ spherical collapse approximates halo size and density contrast;
- ➋ the mass function counts halos of each mass; and
- ➌ the density profile provides the internal structure of halos.

Its power spectrum is the sum of the two-halo power (similar to perturbation theory) and the one-halo power: $\Delta^2(k) = \Delta_{2h}^2 + \Delta_{1h}^2$.

Spherical collapse

Einstein-de Sitter universe [only CDM+baryons (CB)]

Linear spherical overdensity $\delta_0 = \rho_{\text{CB}}/\bar{\rho}_{\text{CB}} - 1$ today

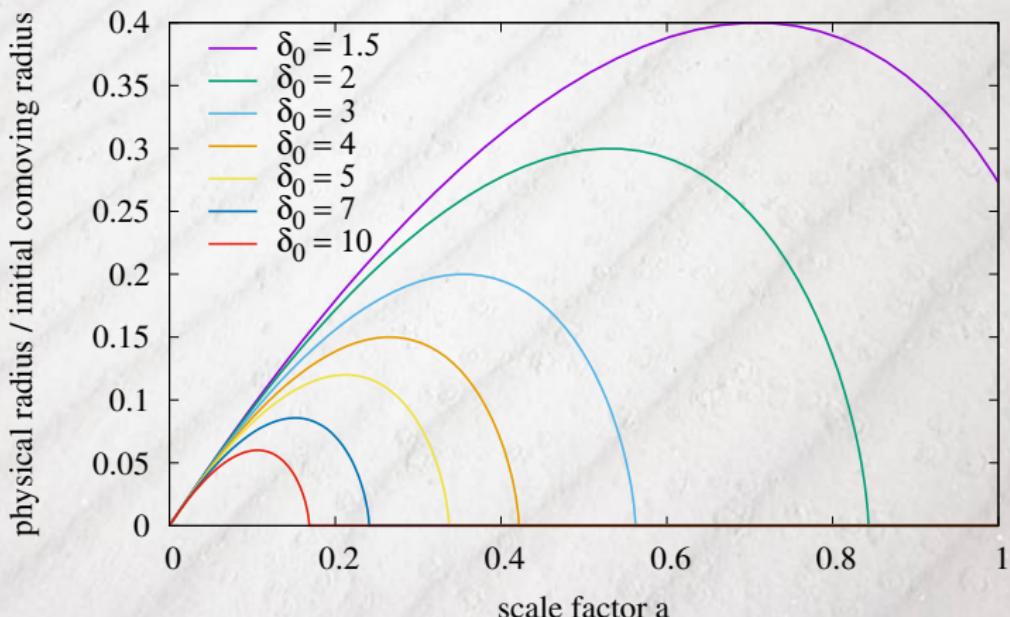
$\Rightarrow a\delta_0$ at scale factor a (fraction a of today's size)

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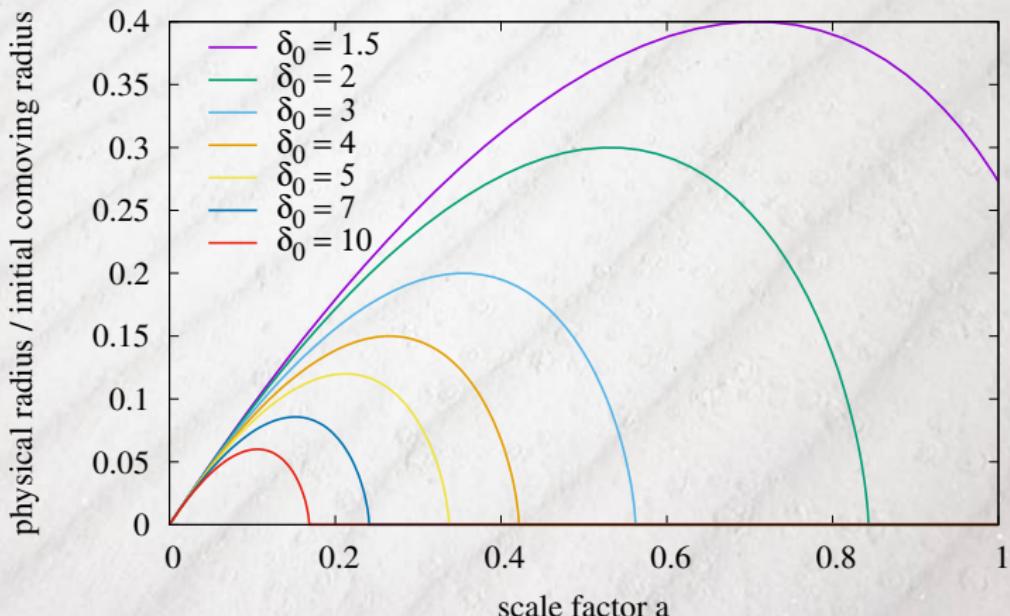
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critical linear density: $\delta_{\text{sc}} = 1.686$



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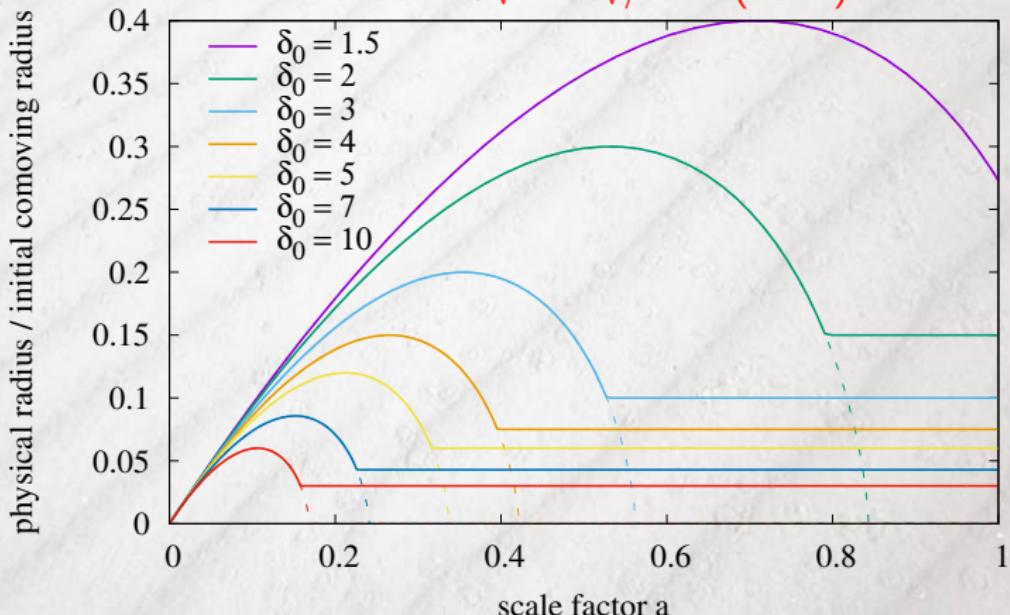
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virial radius ratio: $\mathcal{V}_v = \mathcal{R}_v/\mathcal{R} = (18\pi^2)^{-1/3}$



Mass function

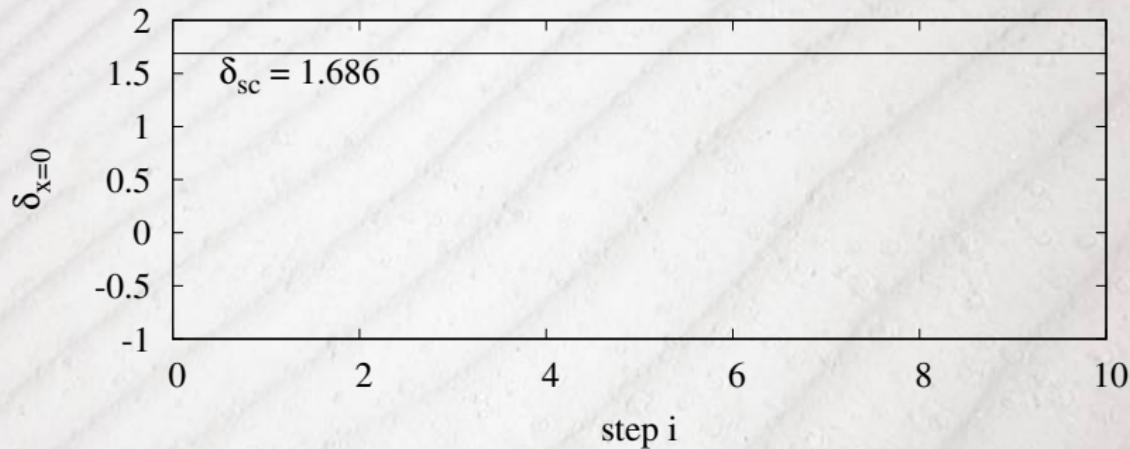
overdensity right here: $\delta_{\vec{x}=\vec{0}} = \int \frac{d^3 k}{(2\pi)^3} \delta(\vec{k})$

split integral so that $V_{\text{univ}} \int_{k_i}^{k_{i+1}} \frac{d^3 k}{(2\pi)^3} = 1 \quad \Rightarrow \delta_{\vec{x}=\vec{0}} \approx \frac{1}{V} \sum_i \delta(\vec{k}_i)$

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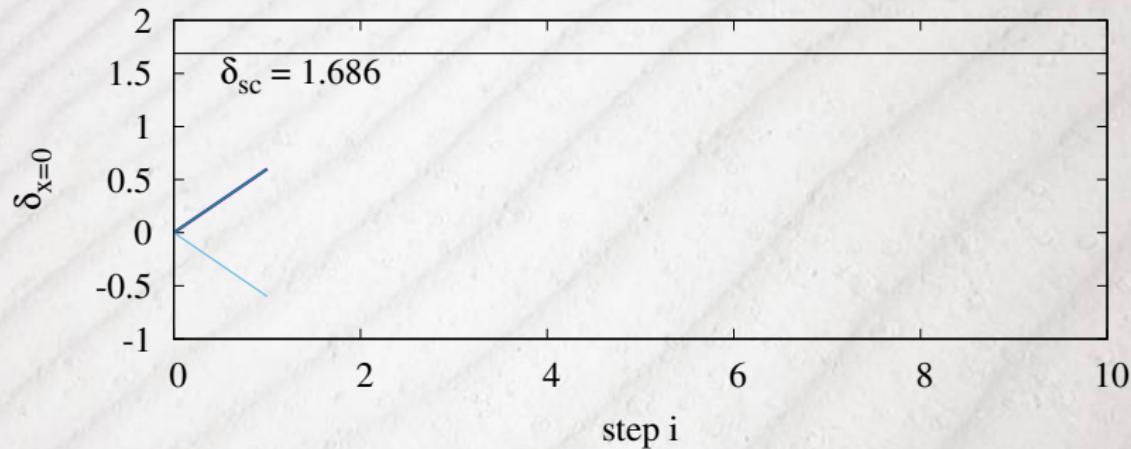
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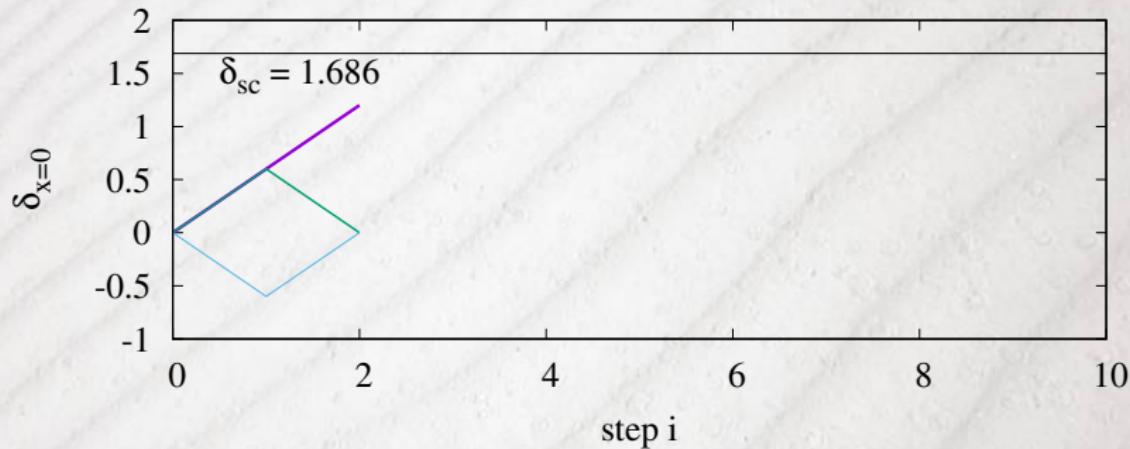
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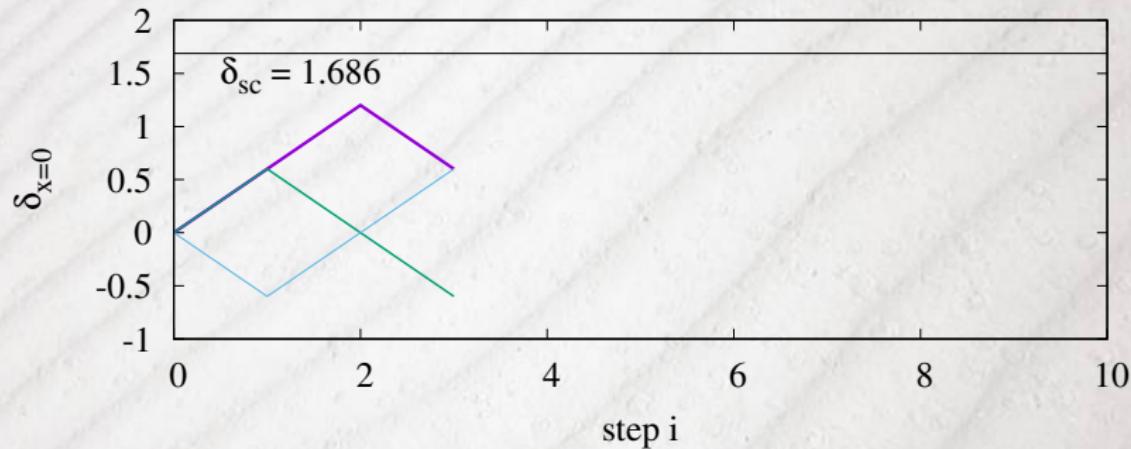
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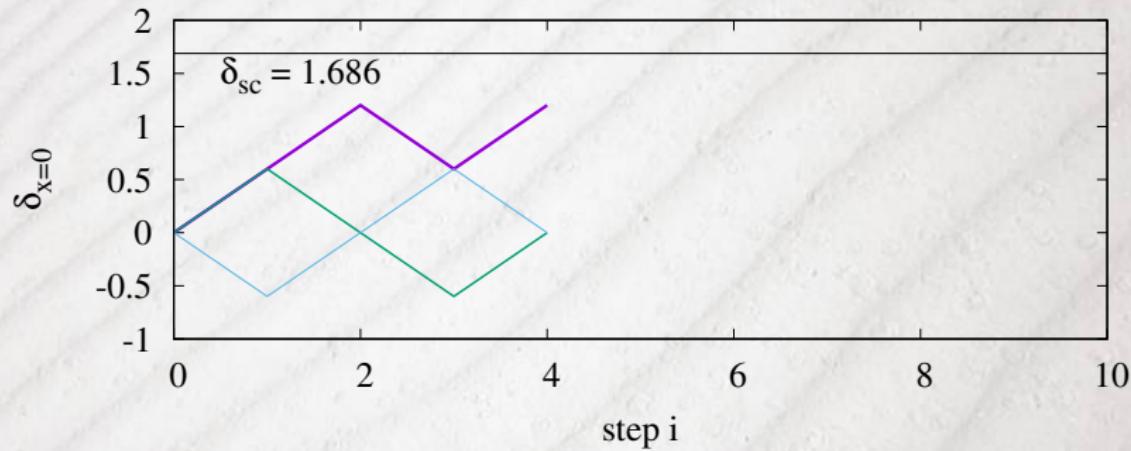
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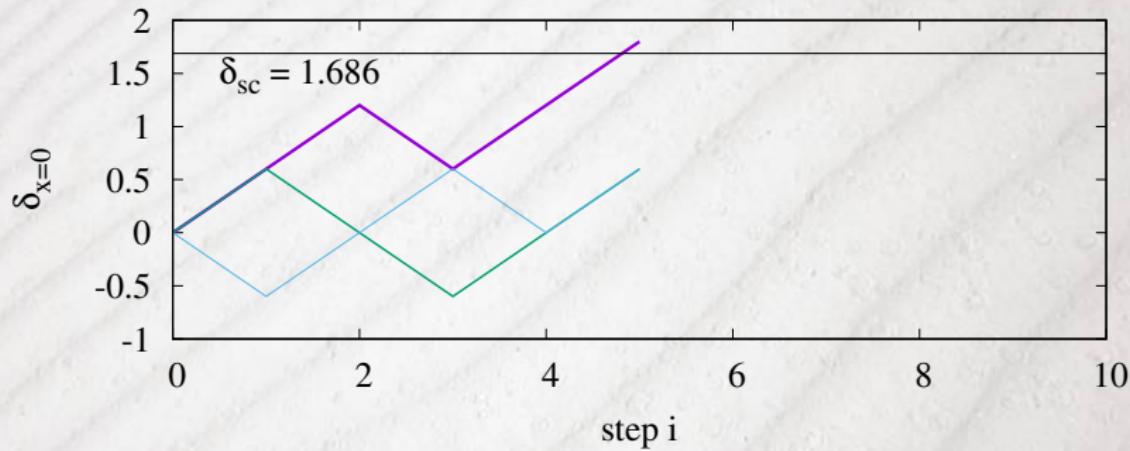
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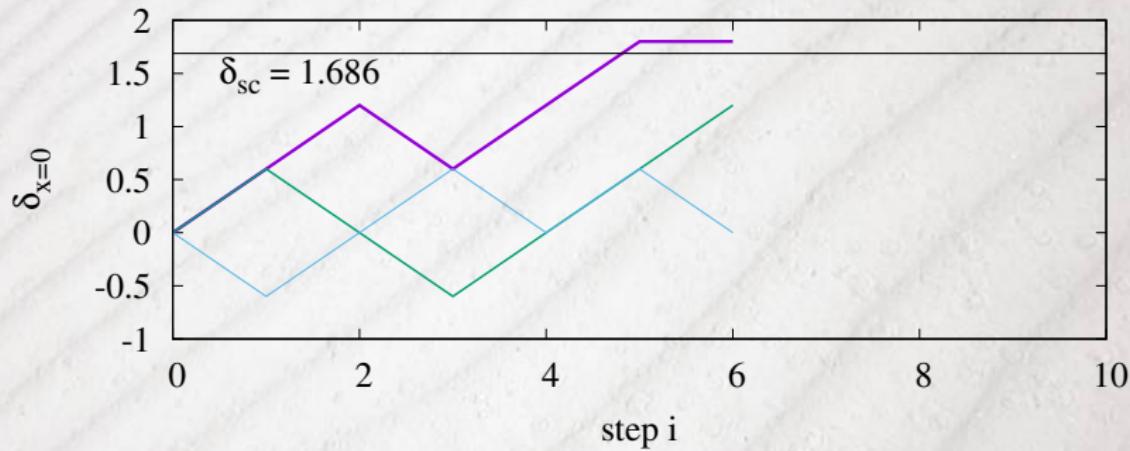
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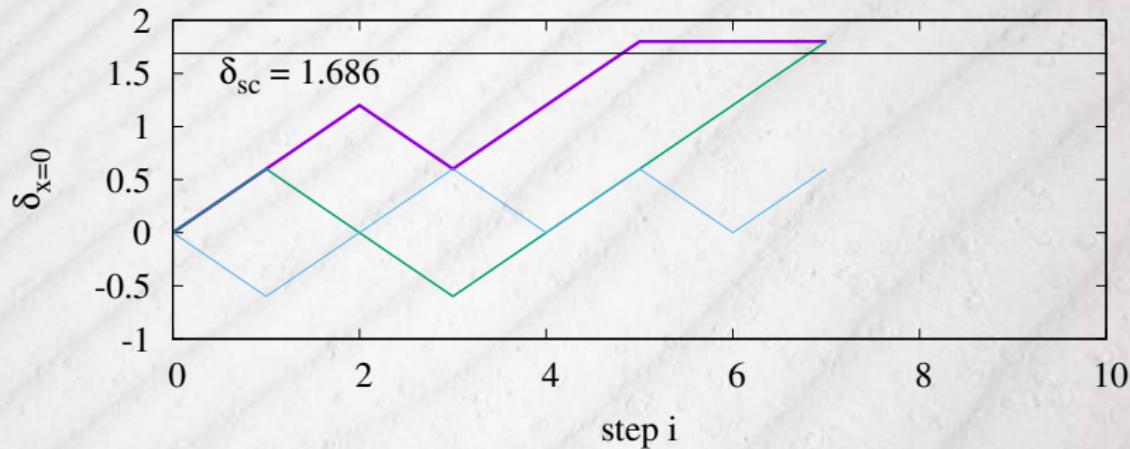
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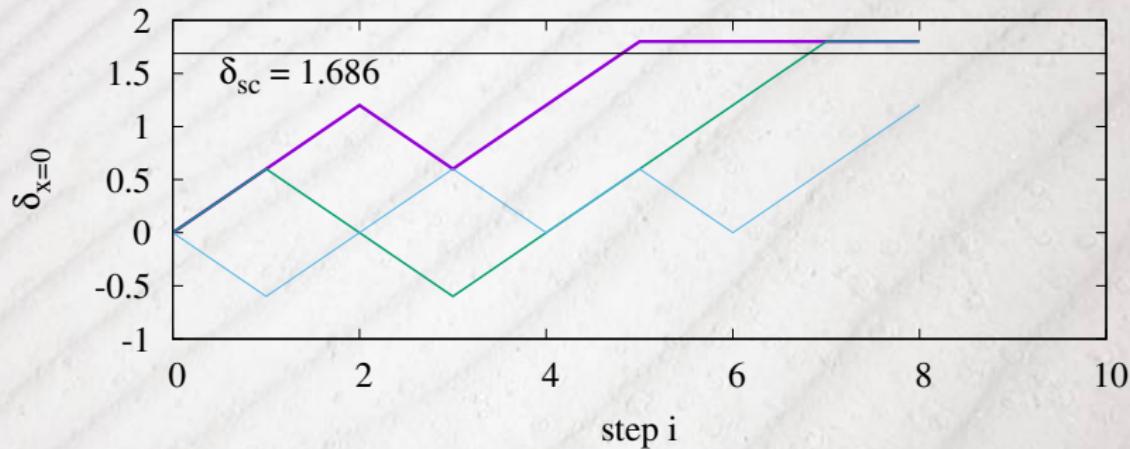
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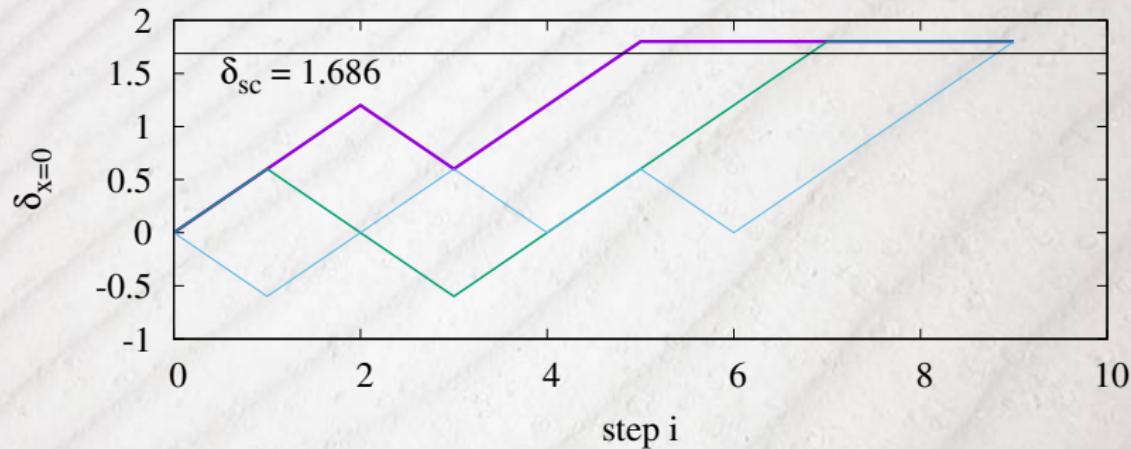
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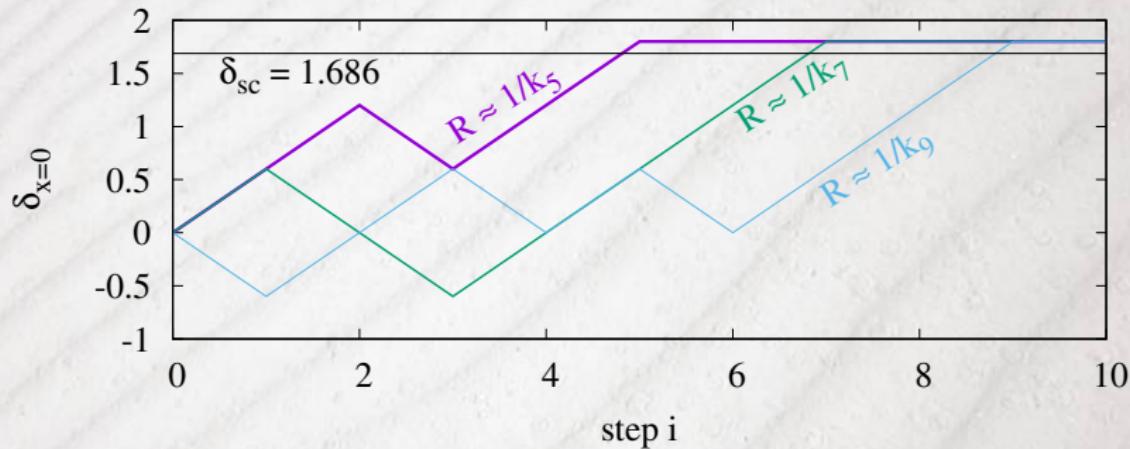
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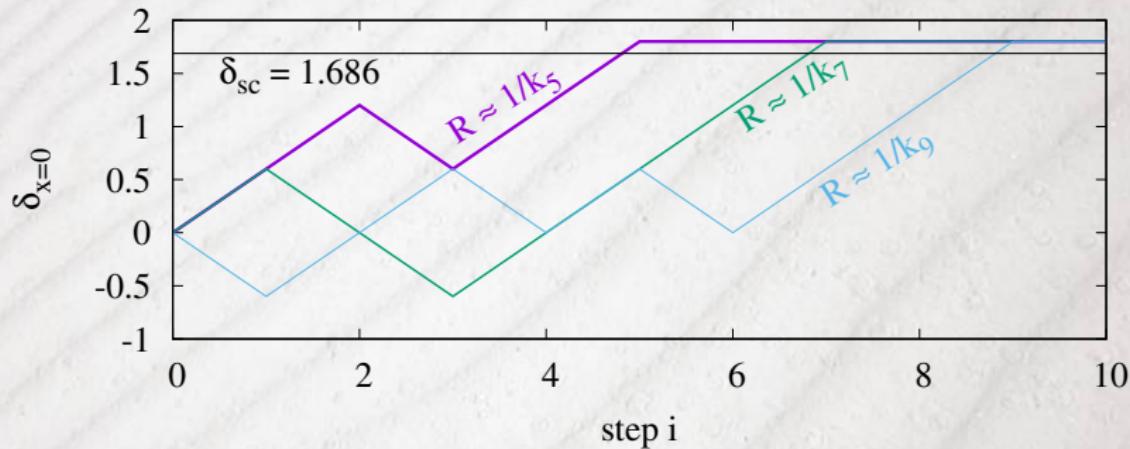
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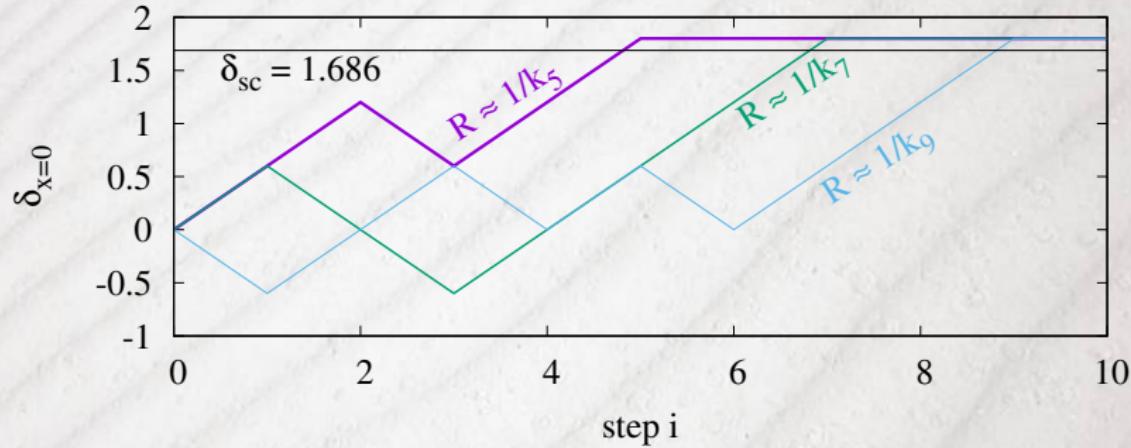


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halo rarity: $\chi(R) = \delta_{sc}/\sigma(R) \Rightarrow R(\chi), M(\chi) = \frac{4}{3}\pi\bar{\rho}_{CB}R(\chi)^3$

Mass function II

We can't predict the halo mass at a particular position, only the probability distribution, which we average to get a number density.

- number density with mass $\leq M$: $n(M)$
- number density with mass $\in [M, M + \Delta M]$: $\frac{dn}{dM} \Delta M$

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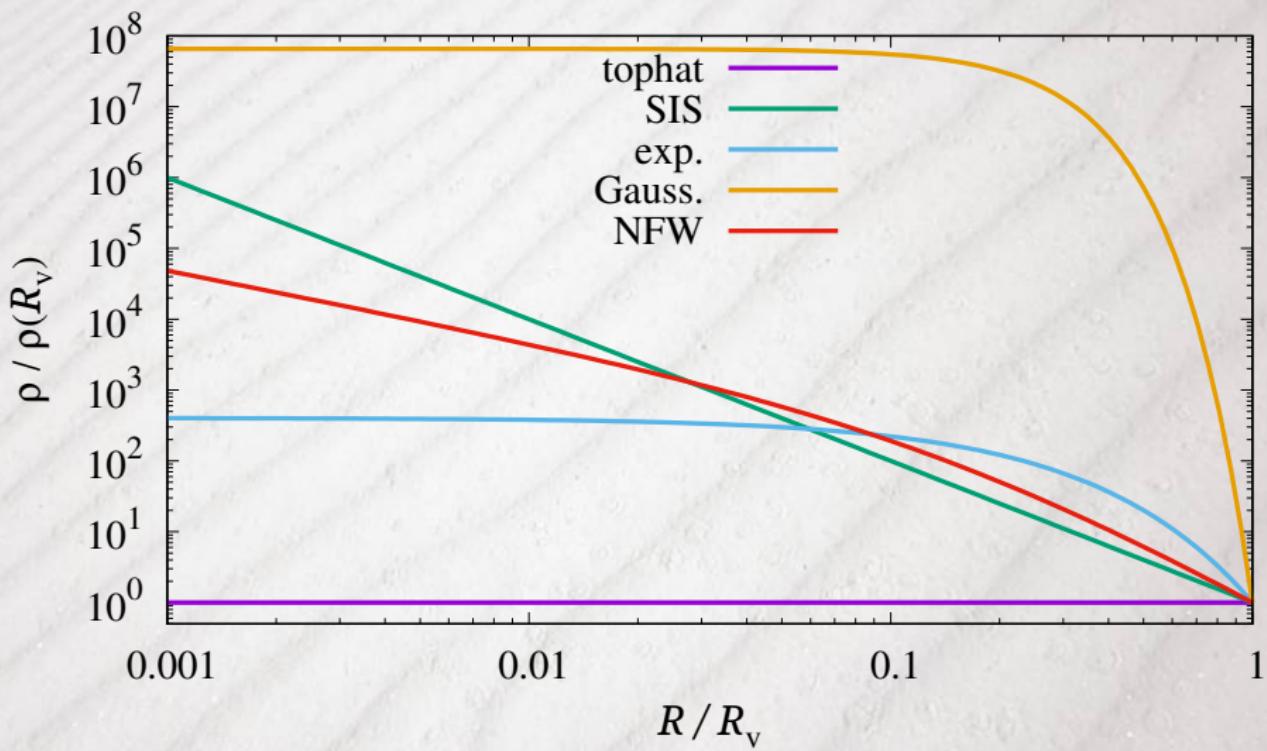
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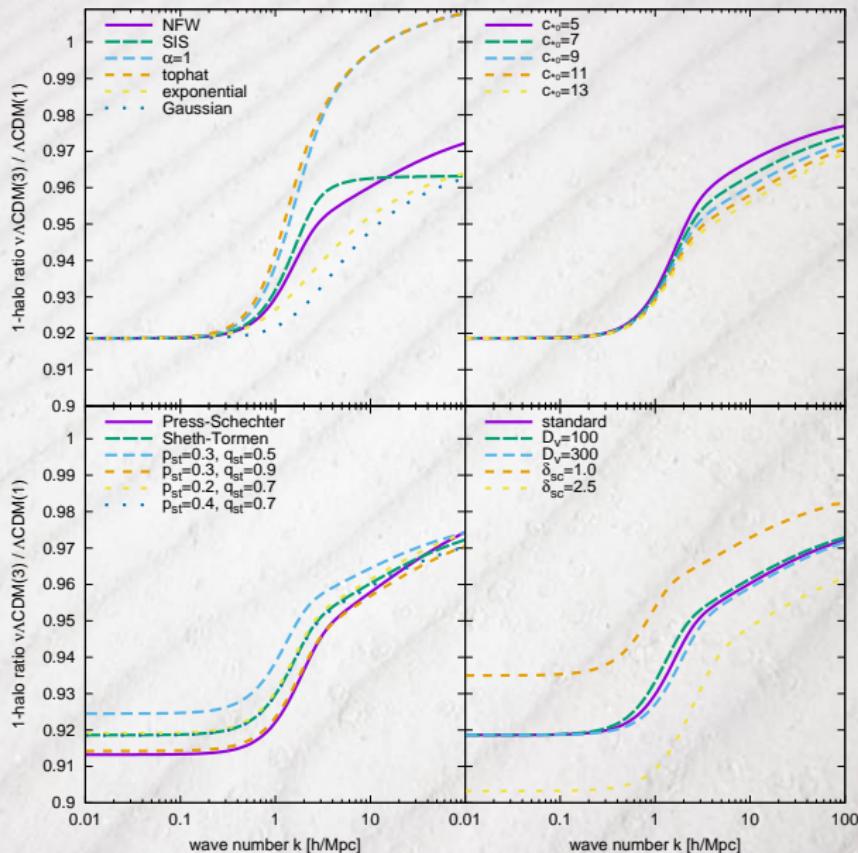
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$f(\chi) = A \left(1 + \frac{1}{(q_{\text{st}} \chi^2)^{p_{\text{st}}}} \right) \exp(-q_{\text{st}} \chi^2 / 2)$ with $p_{\text{st}} = 0.3$, $q_{\text{st}} = 0.707$
Sheth, Mo, Tormen (2001); Sheth, Tormen (2002)

Density profile



The one-halo power spectrum rises with k



General behavior of one-halo power spectrum

one-halo power: $\Delta_{1h}^2(k) = \frac{2k^3}{3\pi} \int_0^\infty d\chi f(\chi) R(\chi)^3 U(kR(\chi)/\mathcal{V}_v)^2$

Fourier transform of spherically-symmetric density:

$$U(s) = \frac{4\pi \mathcal{R}_v^3}{M} \sqrt{\frac{\pi}{2s}} \int_0^\infty dy y^{3/2} J_{1/2}(sy) \rho(y \mathcal{R}_v)$$

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Example: SIS ($\rho(r) \propto r^{-2}$) $\Rightarrow U(kR/\mathcal{V}_v) \rightarrow \pi/(2kR/\mathcal{V}_v)$
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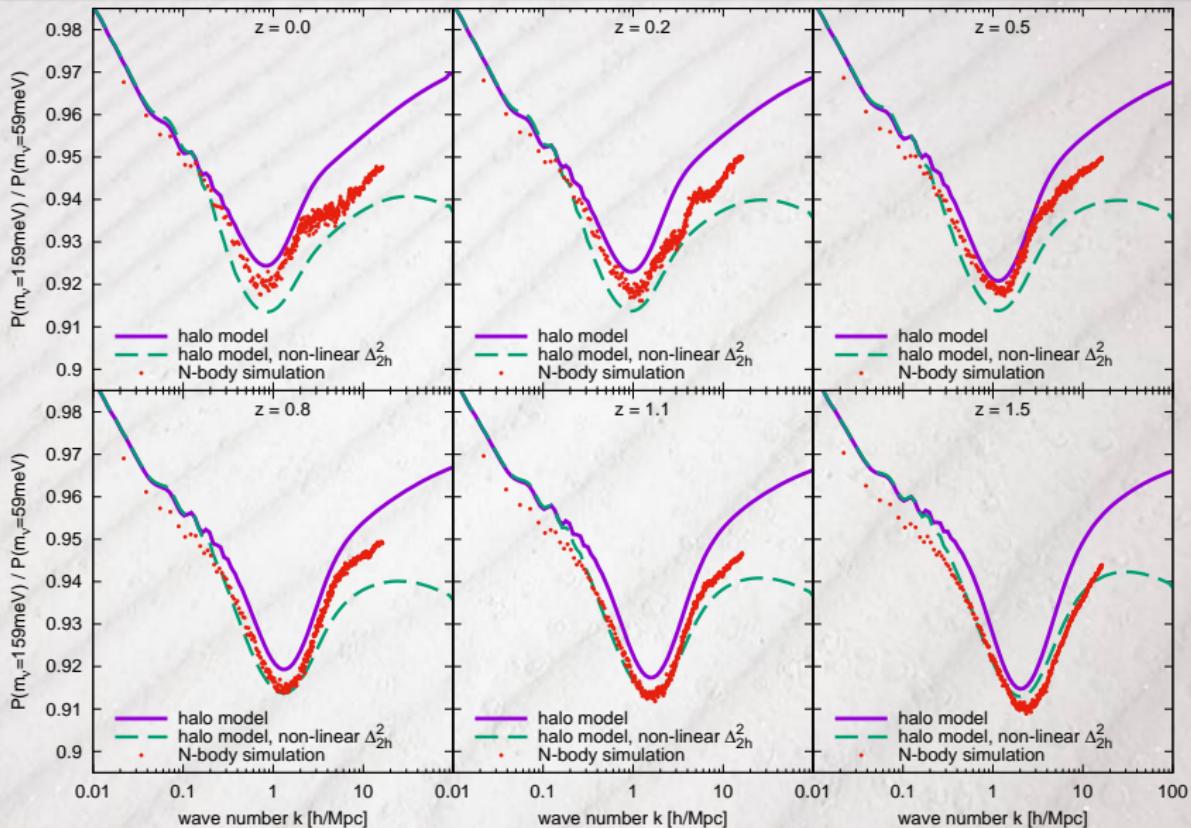
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Halo centers are less cosmology-dependent than halo outskirts.

Spoon: Halo model vs. N-body simulation



Hannestad, Upadhye, and Wong, JCAP 11:062 (2020), arXiv:2006.04995

Conclusions

- ➊ The N-body neutrino spoon is a real, physical feature due to the transition from a two-halo suppression that increases with k to a one-halo suppression that decreases with k .
- ➋ The decreasing one-halo suppression is a general prediction of the halo model, and arises from $U(s)$ switching from flat at low s to declining at high s .
- ➌ With first-principles linear perturbation theory, a standard halo model from 2002 (6 years before the spoon was discovered) predicts the position and redshift-dependence of the spoon.
- ➍ Using non-linear perturbation theory, this halo model also predicts the depth of the spoon for $z \gtrsim 1$.

Hannestad, Upadhye, and Wong, JCAP 11:062 (2020), arXiv:2006.04995